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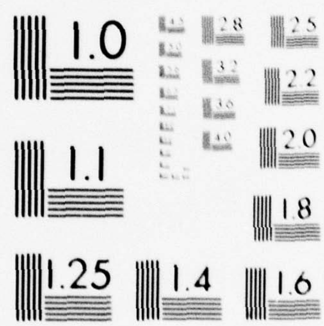

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Multivariate Classes In Reliability Theory

by

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Multivariate Classes In Reliability Theory[1]

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Four classes of lifetimes which have been useful in describing situations where systems are assumed to have independent univariate component lifetimes are: 1) the increasing failure rate (IFR) class; 2) the increasing failure rate average (IFRA) class; 3) the new better than used (NBU) class; and 4) the new better than used in expectation (NBUE) class. These classes are reviewed and also multivariate analogs of the IFR and IFRA cases are discussed. New multivariate definitions of NBU and NBUE are introduced.

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1. Introduction. In many situations where the lifetimes of systems and components are considered, these lifetimes do not have exponential distributions. Rather, these lifetimes reflect the effect of wearout. Four alternative classes of lifetimes which describe various types of wearout and have been extensively studied are: 1) the increasing failure rate (IFR) class; 2) the increasing failure rate average (IFRA) class; 3) the new better than used (NBU) class; and 4) the new better than used in expectation (NBUE) class. In the case where the component lifetimes can be assumed to be independent these classes have proven to be very useful. Recently in an attempt to describe the more realistic situations where this independence assumption cannot be made, various multivariate versions of the above classes have been proposed.

In Section 2 we shall discuss the univariate classes and their properties. Then we shall discuss the most important multivariate analogs of the IFR (Section 3) and the IFRA (Section 4) classes. Properties of these classes as well as their relation to other proposed classes will be given. In Section 5, new multivariate definitions of NBU and NBUE classes will be given and these will be discussed.

Throughout the paper, the terminology and notation of Barlow and Proschan [3] will be used with the exception that a structure function $\phi(\underline{x})$ will be called monotone if it is nondecreasing in each of its components and in addition $\phi(\underline{0}) = 0$ and $\phi(\underline{1}) = 1$.

2. Univariate Classes. The most important and most studied class of lifetimes which describe wearout is the class of distributions with increasing failure rate. This concept has been used in actuarial science, in statistics, and in engineering reliability where it is sometimes called "increasing hazard rate".

Let T be a random lifetime with distribution function $F(x) = P\{T \leq x\}$ and

$F(0) = 0$*, survival function $\bar{F}(x) = 1 - F(x)$ and density $f(x)$ (if it exists). Then T (or F) is said to have increasing failure rate (IFR) if

$$\frac{P\{x < T \leq x + t\}}{P\{x < T\}} \text{ increases in } x \geq 0 \text{ for } t \geq 0. \tag{2.1}$$

This means that, given that T has survived beyond time x, the probability that it fails within the next t units of time increases as x increases, i.e. the older the lifetime, the more likely it is to fail. A lifetime is said to have decreasing failure rate (DFR) if the quantity in (2.1) decreases in $x \geq 0$ for $t \geq 0$. The definition of IFR, more often encountered, which is an instantaneous version of (2.1) and is equivalent to it if the density exists, is that the failure rate function

$$r(x) = \frac{f(x)}{\bar{F}(x)} \text{ increases in } x \geq 0. \tag{2.2}$$

A corresponding statement holds for DFR. Another way of writing (2.1), which is useful in the multivariate cases to be described, is that

$$\frac{\bar{F}(x + t)}{\bar{F}(x)} = \frac{P\{T > x + t\}}{P\{T > x\}} = 1 - \frac{P\{x < T \leq x + t\}}{P\{x < T\}} \text{ decreases in } x \geq 0 \text{ for } t \geq 0. \tag{2.3}$$

This amounts to saying that the survival rate decreases as the age of the component increases. Similarly an increasing version of (2.3) (i.e. increasing survival rate) is equivalent to DFR. Still another version of IFR (DFR) is that the function

$$\log \bar{F}(t) \text{ is concave (convex).} \tag{2.4}$$

The classes of IFR and DFR lifetimes satisfy the following properties:

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*The usual assumption is that $F(0^-) = 0$. We make the simplifying assumption $F(0) = 0$ here for the purpose of this exposition. It leads to slightly simpler definitions.

i) the only lifetimes which are both IFR and DFR are the exponential lifetimes; ii) convolutions of IFR lifetimes are IFR (this means that for independent IFR lifetimes, if a failed component is replaced by a spare, then the accumulated lifetime is IFR). See Barlow and Proschan [3] for additional properties and discussion.

The second major class of lifetimes we will discuss is the class whose lifetimes have <u>increasing failure rate average</u> (IFRA). These are the lifetimes whose survival function satisfies

$$\overline{F}(\alpha t) \geq \overline{F}^{\alpha}(t) \text{ for all } 0 < \alpha \leq 1 \text{ and all } t \geq 0. \qquad (2.5)$$

If the density exists it is not hard to show that this is equivalent to the more intuitive condition that

$$\frac{\int_0^t r(x)dx}{t} \quad \text{increases in } t \geq 0 \qquad (2.6)$$

where $r(x)$ is the failure rate function of (2.2), and so (2.6) gives that the average failure rate increases. Another equivalent form of IFRA, which was used by Block and Savits [5] to establish that the convolution of IFRA lifetimes is IFRA, is that

$$\int_0^\infty h^{\alpha}(x/\alpha)\ dF(x) \geq \{\int_0^\infty h(x)\ dF(x)\}^{\alpha}, \text{ all } 0 < \alpha \leq 1 \qquad (2.7)$$

and all nonnegative nondecreasing functions h.

The class of IFRA lifetimes which is wider than (i.e. contains) the class of IFR lifetimes satisfies the following properties: i) for monotone systems with independent IFRA lifetimes, the system lifetime is IFRA; ii) the IFRA class is the smallest class containing the exponential distributions which is closed under the formation of coherent systems and limits in distribution (see Ross [24] for an extension to IFRA processes); iii) the lifetime of a device arising as a consequence of a natural nonfatal shock model is IFRA (see Theorem 3.8, p. 94,

Barlow and Proschan [3]); iv) convolutions of IFRA lifetimes are IFRA (see Block and Savits [5]). For other properties of this class including statistical procedures see Barlow and Marshall [2], Barlow [1], and Doksum [13], [14].

Still wider classes of lifetimes are those which are new better than used (NBU) and new better than used in expectation (NBUE). A lifetime T is NBU if the probability that it will survive when new is greater than the probability that it will survive given that it is any other age, i.e.

$$P\{T > x + t \mid T > x\} = \frac{\overline{F}(x + t)}{\overline{F}(x)} \leq \overline{F}(t) = P\{T > t\} \text{ for all } x, t \geq 0. \qquad (2.8)$$

A lifetime is NBUE if an integrated version of (2.7) is satisfied, i.e.

$$\int_t^\infty \overline{F}(x)\, dx \leq \mu\, \overline{F}(t) \text{ for } t \geq 0 \qquad (2.9)$$

where $\mu = \int_0^\infty \overline{F}(x)\, dx$ is assumed to be finite.
Another way of expressing that a lifetime T is NBUE is that

$$E(T - t \mid T > t) \leq \mu \quad \text{for} \quad t \geq 0 \qquad (2.10)$$

which means that given any fixed age the residual mean lifetime is smaller than the mean lifetime.

Both the NBU and the NBUE classes have been shown to be useful in the solution of certain maintenance problems. In particular, lifetimes are NBU if replacement policies are to be beneficial in a certain sense. The NBUE lifetimes are the largest class for which the number of failures observed in a "replace at failure only" policy is larger stochastically in the long run then when the process starts. See Chapter 6 of Barlow and Proschan [3] for details. Another situation where NBU arises is given by Ross [23] who shows that for a monotone system with independent, exponential components and exponential repair times, the time until first system failure is NBU. Furthermore, monotone systems

with independent NBU components have NBU lifetimes and convolutions of NBU (NBUE) lifetimes are NBU (NBUE). Tests for NBU and NBUE have been given by Hollander and Proschan [18], [19].

3. Multivariate IFR. There have been several different multivariate extensions of the concept of increasing failure rate. Most of these have been based on one of two notions. The first of these arises from a generalization of (2.3) and a second from a generalization of (2.4).

Let $\underline{T} = (T_1, \ldots, T_n)$ be a multivariate lifetime and let $\overline{F}(\underline{t}) = \overline{F}(t_1, \ldots, t_n) = P\{T_1 > t_1, \ldots, T_n > t_n\}$ be the survival function. Also assume that $\overline{F}(\underline{0}) = 1$ where $\underline{0} = (0, \ldots, 0)$. Since (2.3) is interpreted as a device being less likely to survive as it ages, a multivariate generalization of this is that

$$\frac{\overline{F}(\underline{x} + \underline{t})}{\overline{F}(\underline{x})} \text{ decreases in } \underline{x} \geq \underline{0} \text{ for all } \underline{t} \geq 0. \tag{3.1}$$

This condition, however, is ambiguous since $\underline{x}$ can increase in various ways and $\underline{t}$ can be restricted in certain ways. Several versions of (3.1) will now be given. It is also assumed that similar conditions hold for all marginal distributions. Here $\underline{1} = (1, \ldots, 1)$.

(a) $\dfrac{\overline{F}(x\underline{1} + t\underline{1})}{\overline{F}(x\underline{1})}$ decreases in $x \geq 0$ for all $t \geq 0$.

(b) $\dfrac{\overline{F}(x\underline{1} + \underline{t})}{\overline{F}(x\underline{1})}$ decreases in $x \geq 0$ for all $\underline{t} \geq \underline{0}$.

(c) $\dfrac{\overline{F}(\underline{x} + t\underline{1})}{\overline{F}(\underline{x})}$ decreases in $\underline{x} \geq \underline{0}$ for all $t \geq 0$.

(d) $\dfrac{\overline{F}(\underline{x} + \underline{t})}{\overline{F}(\underline{x})}$ decreases in $\underline{x} \geq 0$ for all $\underline{t} \geq 0$.

Another version in a similar spirit, although not exactly in the same form as (3.1) is

(e) for $i = 1,\ldots, n$, $\dfrac{\overline{F}((x_i + t)_i, \underline{x})}{\overline{F}(\underline{x})}$ decreases in $x_i \geq 0$ for all $t \geq 0$ and all $x_j \geq 0$, $j \neq i$.

Here the notation $((x_i + t)_i, \underline{x})$ means $(x_1,\ldots, x_{i-1}, x_i + t, x_{i+1},\ldots, x_n)$. Version (c) is due to Harris [17] and to Brindley and Thompson [11]; (a), (b) and (d) are due to Marshall [21]; and (e) (in another form) is due to Johnson and Kotz [20]. Many other versions are possible, but it appears that of all of these (c) is perhaps the most important. One reason for this is that (c) captures the idea of the original model, in that time for the components runs at the same rate (i.e. $t = t_1 = \cdots = t_n$). Furthermore the different $x_1,\ldots, x_n$ allows for the possibility that the components are of different ages, as is often the case in practice. Concept (c) is designated <u>MIFR</u> and is entensively discussed in Barlow and Proschan [3]. A dual concept with increasing replacing decreasing is called <u>MDFR</u>. Other reasons for the importance of MIFR are the following properties.

(i) A univariate MIFR lifetime is IFR.

(ii) The union of independent sets of MIFR lifetimes is MIFR.

(iii) The marginal lifetimes of MIFR lifetimes are MIFR.

(iv) Series systems formed with MIFR lifetimes are MIFR.

(v) A lifetime is MIFR and MDFR if and only if it has the multivariate exponential distribution known as the MVE (see Chapter 5 of Barlow and Proschan [3]).

The second notion upon which other versions of multivariate IFR has been based is a generalized form of (2.4). These versions were given by Marshall [21] and involve the idea that $\log \overline{F}(x_1,\ldots, x_n)$ is concave in some sense. This

concavity can be taken in the usual sense or along various curves and lines in R^n. The properties of the various classes which emerge have been briefly discussed by Marshall [21].

4. Multivariate IFRA. The central theoretical role played by the univariate IFRA class derives from the properties satisfied by this class (see i) - iv) following (2.7)) rather than from the fact that it has an increasing average failure rate. It is not surprising then, that various multivariate versions of IFRA have been defined using a generalization of one of these basic properties. The classes of lifetimes which result from these generalizations, however, all fail to satisfy some basic property. Block and Savits [8] have proposed a class of multivariate IFRA lifetimes which is based on a mathematical property of univariate IFRA lifetimes. This class contains a rich variety of multivariate lifetimes and also satisfies all of the fundamental properties which one would expect for a multivariate IFRA class.

The class of multivariate lifetimes proposed by Block and Savits is given in the following definition which is a generalization of the univariate property (2.7).

Definition 4.1. Let $\underline{T} = (T_1, \ldots, T_n)$ be a nonnegative random lifetime. The random vector $\underline{T}$ is said to be MIFRA if

$$E^{\alpha}\,[h(\underline{T})] \leq E[h^{\alpha}(\underline{T}/\alpha)] \qquad (4.1)$$

for all continuous nonnegative nondecreasing functionsh and all $0 < \alpha < 1$.

Other conditions which have been proposed for multivariate IFRA are given in the next definition. As in the previous material, we rely heavily on the notation and terminology of Barlow and Proschan [3] with the one exception mentioned in Section 1. The life function τ corresponding to a structure function ϕ is called monotone (coherent) if ϕ is monotone (coherent). See Esary and

Marshall [15] for a discussion of life functions.

Definition 4.2. Let $\underline{T} = (T_1, \ldots, T_n)$ be a nonnegative random lifetime with survival function $\overline{F}(\underline{t}) = P\{\underline{T} >> \underline{t}\}$ where ">>" means strict inequality holds for each component. The vector $\underline{T}$ is said to satisfy condition __ if the condition following letter __ is satisfied:

A: $\overline{F}^{\alpha}(\underline{t}) \leq \overline{F}(\alpha\underline{t})$ for all $0 < \alpha \leq 1$ and all $\underline{0} \leq \underline{t}$.

B: $\underline{T}$ is such that each monotone system formed from $\underline{T}$ is univariate IFRA.

C: $\underline{T}$ is such that there exist independent IFRA random variables $X_1, \ldots, X_k$ and monotone life functions τ_i, $i = 1, \ldots, m$ such that $T_i = \tau_i(X_1, \ldots, X_k)$ for $i = 1, \ldots, m$.

$\sum$: $\underline{T}$ *is such that* there exist independent IFRA random variables $X_1, \ldots, X_k$ and nonempty sets S_i of $\{1, \ldots, k\}$ such that $T_i = \sum_{j \in S_i} X_j$ for $i = 1, \ldots, m$.

D: $\underline{T}$ is such that there exist independent IFRA random variables $X_1, \ldots, X_k$ and nonempty subsets S_i of $\{1, \ldots, k\}$ such that $T_i = \min_{j \in S_i} X_j$ for $i = 1, \ldots, m$.

E: $\underline{T}$ is such that the minimum of any subfamily of $T_1, \ldots, T_m$ is IFRA.

F: $\underline{T}$ is such that $\min_i a_i T_i$ is IFRA for all $a_i \geq 0$, $i = 1, \ldots, m$.

Conditions A,B,C,D,E,F have been given by Esary and Marshall [16] and condition $\sum$ was given by Block and Savits [9].

The following relationships hold between MIFRA and the seven conditions (see Block and Savits [10] for proofs and examples).

$$\begin{tikzcd}
D \arrow[d, Rightarrow] & & \Sigma \arrow[ddl, Rightarrow] \\
C \arrow[dr, Rightarrow] & & \\
& \mathrm{MIFRA} \arrow[dl, Rightarrow] \arrow[dr, Rightarrow] & \\
A(=)F \arrow[dr, Rightarrow] & & B \arrow[dl, Rightarrow] \\
& E &
\end{tikzcd}$$

Figure 1

With the exception of the implication $\Sigma \Rightarrow C$ (which is conjectured to be false), Block and Savits [10] have shown that no other implications are possible.

The following properties are basic for a class of multivariate IFRA distributions.

(P1): Closure under the formation of monotone systems.

(P2): Closure under limits in distribution.

(P3): Marginals are in the same class.

(P4): Closure underconjunction of independent lifetimes.

(P5): Closure under scaling.

(P6): Closure under convolution.

It has been shown by Block and Savits [8] that the MIFRA class satisfies all six of these. Furthermore in a subsequent paper, Block and Savits [10] have demonstrated that each of the seven conditions in Definition 4.2 fail to satisfy at least one of these properties. Specifically A, Σ, E and F fail to satisfy (P1); B and C fail to satisfy (P5); and D fails to satisfy (P6).

5. Multivariate NBU and NBUE

Unlike the previous two multivariate classes discussed, only preliminary work has been done in the situation of multivariate NBU and NBUE classes. We shall present 12 possible definitions of NBU and a similar group for NBUE.

The only versions of multivariate NBU of which we are aware in the literature have been discussed by Buchanan and Singpurwalla [12] and Marshall and Shaked [22]. These generalize (2.7), have a similar interpretation, and are related to condition (3.1). They specify that for the random lifetime $\underline{T} = (T_1, \ldots, T_n)$.

$$P\{\underline{T} >> \underline{x} + \underline{t} \mid \underline{T} >> \underline{x}\} = \frac{\overline{F}(\underline{x} + \underline{t})}{\overline{F}(\underline{x})} \leq \overline{F}(\underline{t}) = P\{\underline{T} >> \underline{t}\} \tag{5.1}$$

for various choices of $\underline{x}$ and $\underline{t}$. Conditions 1) - 3) in the following are the ones given by Buchanan and Singpurwalla. Condition 4) is of the same type and was given by Marshall and Shaked [22] and Condition 5) is similar in spirit to e) of Section 3.

1) $\overline{F}((x + t)\underline{1}) \leq \overline{F}(x\underline{1})\, \overline{F}(t\underline{1})$ for all $x \geq 0$, $t \geq 0$.
2) $\overline{F}(x\underline{1} + \underline{t}) \leq \overline{F}(x\underline{1})\, \overline{F}(\underline{t})$ for all $x \geq 0$, $\underline{t} \geq 0$.
3) $\overline{F}(\underline{x} + \underline{t}) \leq \overline{F}(\underline{x})\, \overline{F}(\underline{t})$ for all $\underline{x} \geq 0$, $\underline{t} \geq 0$.
4) $\overline{F}(\underline{x} + \underline{t}) \leq \overline{F}(\underline{x})\, \overline{F}(\underline{t})$ for all $\underline{x} \geq 0$, $\underline{t} \geq 0$ which satisfy $(x_i - x_j)(t_i - t_j) \geq 0$ for all i, j.
5) $\overline{F}(\underline{x} + t\varepsilon_i) \leq \overline{F}(\underline{x})\, \overline{F}(\varepsilon_i)$ for all $\underline{x} \geq 0$ and $i = 1, \ldots, n$ where $\varepsilon_i = (0, \ldots, 0, 1, 0, \ldots, 0)$ and 1 appears in the ith position.

The next condition is a variant of these and actually specifies several conditions depending on the class of life functions chosen.

6) $P\{\tau(\underline{T} - \underline{x}) > t \mid \underline{T} > \underline{x}\} \leq P\{\tau(\underline{T}) > t\}$ for all $t \geq 0$, $\underline{x} \geq 0$ and all τ in a class of life functions (e.g. for all minimums).

If the class of life functions specified is the class of minimums (i.e. series systems) then 6) can be interpreted as the probability of survival for more than t units of any series system formed from a set of components of age at least $\underline{x}$

being less than the probability that the corresponding new series system survives more than t units.

Conditions 7) - 9), to be given, are similar in spirit to the conditions B) - F) in Section 4.

7) $\tau(T_1, \ldots, T_n)$ NBU for all τ in a certain class of life functions (e.g. for all monotone functions, all minimums, all sums).

8) There exist independent NBU $X_1, \ldots, X_k$ and monotone life functions τ_i, $i = 1, \ldots, n$ such that $T_i = \tau_i(\underline{X})$ for $i = 1, \ldots, n$.

9) $\tau(a_i T_i)$ NBU for all $a_i \geq 0$ and all τ in a certain class of life functions.

Again each of 7) - 9) represent several conditions depending on the specification of the τ_i or τ.

The following condition is the generalization of a characterization of NBU given in Block and Savits [7].

10) $E(h(\underline{T} - \underline{x}) \mid \underline{T} > \underline{x}) \leq E(h(\underline{T}))$ for all nonnegative, nondecreasing and continuous h.

Another possible generalization of NBU is through the concept of multivariate shock models of a type discussed in the univariate case by Barlow and Proschan [3]. Several multivariate shock models have been discussed by Marshall and Shaked [22] and by Block [4]. Using the notation of Section 3.0 of the latter paper we have

11) $\underline{T} = (\sum_{i=1}^{N} X_{i1}, \ldots, \sum_{i=1}^{N} X_{in})$ where $(X_{i1}, \ldots, X_{in})$ has independent exponential marginals and these vectors are independent for $i = 1, 2, \ldots$ and N is univariate NBU and independent of the X_{ij}.

There are many variants of this.

A final version of multivariate NBU can be given by generalizing a characterization of NBU given by Block and Savits [6] and using the Laplace transform. This involves giving a discrete condition analogous to 1) - 5) on coefficients related to the Laplace transform of a random lifetime $\underline{T}$. Since this is notationally involved we will omit details.

Comments on various possible NBUE definitions can similarly be made. Various integrated versions of 1) - 6) can be given. Four versions of 1) - 3) are given by Buchanan and Singpurwalla [12]. Versions of 7) - 9) can be given where the τ and τ_i are specifically taken to be sums. Since a characterization of NBUE is given in Block and Savits [7] a multivariate version analogous to 10) can be given. If in 11) N is assumed to be NBUE instead of NBU a multivariate NBUE definition is obtained. Furthermore the comments in the paragraph following 11) apply as well to the NBUE case.

As mentioned previously, little work has been done on multivariate NBU and NBUE classes. Determining which definitions are most fundamental and how these concepts are related to one another still remains to be done. This problem is currently being studied by the authors.

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